DOCUMENT RESUME

ED 265 112 SP 026 870

AUTHOR Hill, Donald M.
TITLE Enhancing the International Dimension of Undergraduate Teacher Education (Mathematics): Vox Clamantis in Deserto?
SPONS AGENCY American Association of Colleges for Teacher Education, Washington, D.C.
PUB DATE 2 Mar 85
NOTE 28p.; Paper commissioned by the American Association of Colleges for Teacher Education for the Guidelines for International Teacher Education Project. For related documents, see SP 026 860-872 and SP 026 874.
PUB TYPE Viewpoints (120) -- Reports - Descriptive (141)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Global Approach; Higher Education; *International Education; International Educational Exchange; *Mathematics Education; Mathematics Teachers; *Preservice Teacher Education

ABSTRACT

Now is the time for educators to strengthen commitment to "global awareness" in colleges of education, professional organizations, the schools, and society itself. Ways to enhance the learning and teaching of mathematics by adding an international dimension are discussed. Areas which appear promising for the addition of an international dimension include: history of mathematics; comparative studies; word problems and applications; metrification; mathematical games; mathematics contests; and computers and technology. This dimension can be added through such instruments as: courses; foreign journals; materials from professional organizations; curricula; foreign visitors, students, and teachers; study abroad; and professional publications. (CB)

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Enhancing the International Dimension of Undergraduate Teacher Education(Mathematics): Vox Clamantis in Deserto?

Donald M. Hill, Ph.D.
Professor of Mathematics
Florida A&M University
Tallahassee, FL 32307
(904) 599-3595

March 2, 1985

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This working paper was prepared for the Guidelines for International Teacher Education Project of the American Association of Colleges for Teacher Education(AACTE).

SP026870

SUMMARY AND GUIDE TO "INTERNATIONAL DIMENSION IN MATH"

Don Hill

Summary:

In spite of the myriad difficulties the mathematical community now has in producing math teachers, mathematics can play a supporting role in promoting global awareness, provided that we are one component of a total effort across all disciplines. An external force causing us mathematics educators to seek ways by which an international dimension may be added to undergraduate teacher education would be helpful and probably necessary. If such comes about through your efforts, our professional organizations could appoint a committee to explore and develop some of the ideas raised in this paper. The most promising are an emphasis on the inherent international nature of mathematics, courses in the history of mathematics, and "comparative" studies. These would not disrupt recently developed guidelines/recommendations by NCTM and MAA.

Guide:

Various warnings, concerns, questions, doubts, objections, etc., are raised as follows:

-- The views in this paper have not been submitted to MAA and NCTM for "official" approval(#2).
-- This paper is not a careful review of the literature. It is based on personal experience and reactions from colleagues(3).
-- Most math educators may not be very enthusiastic about an international dimension(4).
-- "Global awareness" may detract from the preparation of math teachers and the instruction of students(7).
-- We do not have enough math teachers(8), we are asked to do the impossible with crash training(9, 10), what seems like large funding for math/sci is inadequate(9), we have relatively new professional guidelines/recommendations from NCTM and MAA(10), yet we are sadly lacking even by them(11). Implied conclusion: This may not be the time to charge off in a new direction, particularly if it is perceived to hinder what we math educators are trying to accomplish.

Nonetheless, I am positive about global awareness(1, 45, 46), daily see the need of understanding other cultures(6), and feel that you can work with NCTM and MAA(9) to build on our current recommendations(10). A powerful, external factor calling for an international dimension across the board(not just math) would be helpful(12). Our professional organizations(NCTM(13), MAA(14), ICMI(15)) could play a crucial role in collecting, refining, and disseminating information to math educators(12). To my knowledge, such information does not exist, at least not in one place. A committee could carry out this assignment(44). Good contacts are given throughout the paper(particularly 43).

Eight areas are mentioned as vehicles for global awareness(12). The first three are worthy of serious consideration, the fourth looks promising to me but is almost universally panned by prominent math educators, and the last four strain the imagination:

-- Math itself(16). $x^2 - 1 = (x + 1)(x - 1)$ is the same in any high school in the world. The commutative and associative properties for addition of natural numbers are the same regardless of the numeration system used. Likewise the mathematical "inclusive or" means the same to each mathematician no matter what the peculiarities of his mother tounge are. We math educators generally do not at all stress the international character of math, but we could with very little disruption to what we already are doing.

-- History of math(18). A study of the history of math shows the debt we owe other cultures for their contributions to math. Moreover, mathematiciansoften corresponded with each other for the advancement of math even when their governments were enemies. Random glances at the MAA Recommendations(pp 10-13)should convince anyone of the international flavor of such a course. In spite of MAA and NCTM backing, I feel that "history" is not taken by many students. It should be. It is definitely a good candidate for "what is possible."

-- "Comparative studies." Wherever we use domestic sources in teacher education in math there may be a possibility of an appropriate international source. My perception is that this is not done very widely, but could be (28-35).

① Dartmouth College was founded in 1769 with the motto, "Vox Clamantis in Deserto." Some of us with special interest in the teaching of mathematics in other countries have felt, from time to time, that we too could appropriately use my alma mater's motto. I now have reason to be more optimistic than in the past. I strongly believe, and even more fervently hope, that it is now time for all educators to strengthen our commitment to "global awareness" in our colleges of education, our professional organizations, our schools and, ultimately, society itself.

② At this point let me make it perfectly clear that I am writing as an individual and not as a representative of the National Council of Teachers of Mathematics (NCTM) or the Mathematical Association of America (MAA). I do, however, serve as the Liaison for International Mathematics Education of the former, am a member of the Board of Governors of the latter, and, by virtue of chairing the U. S. Commission on Mathematical Instruction of the National Academy of Sciences/National Research Council, was the American representative to the International Commission on Mathematical Instruction which met last August in Australia. Instead, I write as one whose entire professional career in mathematics has been enriched by study in Europe, work in Africa in a variety of capacities spanning three decades through six trips ranging from two weeks to two years, and by 13 years of teaching at a traditionally black university in the South.

③ A further word of caution is in order for the reader. This paper is not an extensive review of the literature. It is based on my own experiences, a sampling of the literature and exchanges with professional friends over the years as well as responses from the 30 individuals I recently contacted about this specific issue. Those contacted include presidents and past presidents of the MAA and NCTM, others known for their leadership roles within those organizations, and finally, a small group of individuals whose mathematical interests are strongly tied to the teaching of mathematics in other countries. I am indebted to them for their comments. The responses ranged from post cards to extensive letters (see attached letter from Jeremy Kilpatrick). Their responses were almost uniformly favorable to the idea of significantly enhancing the international dimension of undergraduate teacher education.

(4) Of real concern to me is the degree to which the rank and file mathematics educators will willingly accept yet another thing to be added to the long list which they already have to do in order to prepare mathematics teachers for the schools of our country. Casual conversation with a couple of these individuals has indicated a pronounced disinterest in "global awareness." However, if a fire alarm were to sound, I assume these mathematics educators and their students would leave the classroom immediately. The point is that occasionally something of overriding importance happens that causes us to rearrange what we are doing. With obvious exaggeration, I place "global awareness" in the same category.

(5) In this paper, I assume that "international education" is so important that it must and will be implemented. Leaving the defense of that statement to AACTE, I will not argue that an international dimension will enhance the learning and teaching of mathematics (although that could be done); rather, I will exhibit ways through which we in the mathematical community can do our part.

(6) Although hardly needed, I cannot resist the temptation to share with you the following letter to the editor of our local newspaper (2/3/85) as an example of the need we Americans have to learn more about and appreciate more deeply contributions from those who differ from the mainstream (also see the letter from Kilpatrick):

> In answer to Betsy Robertson's Jan. 23 column, I would like to request her to speak only for herself and her ancestors when she professes feelings of guilt about slavery in the antebellum United States. As for me, I am proud of the role my ancestors have had in liberating the African Negroes from their own primitive, warring, tribal and superstitious way of life on the dark continent; I believe many in Africa where self-government has been restored to them are not much better off today.
>
> The African slaves learned to think and to weave her beautiful counterpane only because they were taught to do so by the white people. My ancestors on all sides were Southerners and I am proud of the fact that they were responsible for teaching the black people all the skills and trades they themselves had learned and known.
>
> The white South even shared its blood with the black people. Observe the history of Thomas Jefferson. At present that infusion of blood has been a large factor in the successful "blacks" of the present day.
>
> Speak for yourself Betsy, and your ancestors, and leave mine to their own well-earned peace.

> Also Betsy, Kentucky was composed in part of abolitionists, and they all turned coat when they saw who was winning the war. This fact should relieve any feeling of guilt in your or your ancestors.

(7) Another question sure to be raised concerns who determines what "global perspective" our teachers and students will have. In a 1983 School Science and Mathematics article (1), Kromhout and Good wrote:

> Our second point, that the Hofstein-Yager proposal (Societal Issues as Organizers for Science Education in the '80s) is in fact dangerous, not just to the goal of scientific literacy, but indeed to science and to our democratic social structure, is that the organization of science courses by social issues opens science education to manipulation and perversion by social activists into a vehicle for reinforcing one-sided or narrow perceptions of social, political, intellectual, and religious issues. If the reader doubts the reality of this danger, consider the present trends in the content and tone of biology, geology, and astonomy texts for the public schools.

Mutatis mutandis, much the same could be said about "global awareness" and mathematics education. It is a serious question but one that goes far beyond mathematics. Each discipline will have the same question as "global awareness" relates to its content area. Thus, generalists will have to provide the answer.

(8) The last bit of caution I wish to raise concerns the woeful lack of adequate numbers of teachers of school mathematics. The problem has been documented elsewhere ad infinitum and will not be repeated here. Thus it may seem strange that, with all the problems the mathematical community now has in trying to produce sufficient mathematics teachers, I advocate adding yet another "hurdle" -- an international dimension.

(9) As the Peace Corps Coordinator of Mathematics, Science and Teacher Training for French-speaking Africa of the Experiment in International Living's 1977 state-side program (2), I am well aware of the difficulties involved in producing "summer wonders" as mathematics teachers for 9-12. Two years ago, I served on the Speaker's Task Force on Mathematics, Science and Computer Education of the Florida House of Representatives. Most of our recommendations were incorporated almost verbatim into the Statutes of Florida and $40,000,000 was appropriated for improving math/science in Florida (3). Yet, it is amazing how little could be done with that large sum of money. Yes, we have problems in producing math teachers. However, I believe that we

can introduce/expand an international dimension in the preparation of math school teachers without great difficulty by building on the work of NCTM and MAA.

(10) Many of my friends have worked hard over the past five years to produce MAA and NCTM guidelines for mathematics teacher training programs that are consistent with each other and that are excellent in their own right. I do not propose an alternate set of curriculum guidelines for mathematics teacher training programs. Rather, we can use existing programs while strengthening an international dimension. We must push for the strong preparation of K-8 teachers and decry the policy of turning out two-week wonders to meet the needs in 9-12.

(11) Some of my colleagues feel that it would be a mistake to treat the NCTM and MAA recommendations as descriptions of what is actually taking place and they are right. The data we have seen make it very clear that teacher preparation programs that rise to the level represented by these documents are still rare. These friends fear that further efforts <u>now</u> to go beyond those documents would detract from their force as descriptions of ideals toward which most of us should still be working rather than as accounts of the status quo.

(12) Having raised possible objections to an increased role for global awareness in the teaching of school mathematics and the preparation of the teachers, let me now be more positive. The following are areas that appear promising to me:

1. Mathematics (itself)
2. History of mathematics
3. Comparative studies
4. Word problems and applications
5. Metrification
6. Mathematical games
7. Mathematics contests
8. Computers and technology?

Before delving into each of these areas, let me state that I believe an externally-driven motivating factor should be used to promote an international dimension in mathematics education. Let me give an example of the obvious. If NCATE were to generically demand that "...schools of education shall demonstrate an international dimension which permeates...," then mathematics educators and our professional organizations would be motivated to find ways to carry it out. The same would tend to happen in our schools, if the accrediting groups (e.g., SACS for us in Florida) were to make similar statements. The same also goes for the actual certification of teachers in the various states. Instead of possibly explaining why the programs are already too crowded and other assorted excuses, individuals and institutions would turn to promising avenues that have already been developed or are in the process of so being. The NCTM, MAA and the International Commission on Mathematical Instruction (ICMI) could be of great assistance. Because each of these three groups is crucial, let me describe them briefly.

(13) The 40,000-member NCTM is primarily interested in the teaching of mathematics K-14 and teacher education. It has conferences, journals, publications, and committees which effectively promote excellence in mathematics teaching. It bodes well that the very first issue of the first volume of the Mathematics Teacher (September 1908) contained two reports of interest about an international dimension in mathematics education. One concerned the acquisition of historical objects from Japan, China, India and surrounding lands by David Eugene Smith. The other was about the formation of an international commission on mathematical instruction and plans for the preparation of national reports on the teaching of mathematics. By the following year an American committee had been appointed. The list of names and institutional affiliations of those associated with the American report runs around ten pages. Perhaps the forefathers of NCTM were more concerned about

international education than we are today. In more recent years the international work of NCTM was a part of the responsibility of the External Affairs Committee, but with EAC's demise the work is now largely done through a Liaison for International Mathematics Education.

(14) The 20,000-member MAA is primarily interested in mathematics at the undergraduate level, although it has obvious interests in what happens both before and after. It has long demonstrated a concern for the preparation of school mathematics teachers. Its international activities occur primarily through vigorous participation in the International Congresses on Mathematical Education (ICMEs). American Perspectives on the Fifth International Congress on Mathematical Education was published earlier this year by MAA.

(15) ICMI is a commission of the International Mathematical Union. The latter is a group of research mathematicians who hold congresses every four years, the next being in Berkeley in 1986. ICMI organizes the ICMEs which are also held every four years. ICME-5 was held last August in Australia and drew 2,000 of the world's leading mathematics educators from 70 countries, including 251 Americans from 44 states (4). It is the unique forum for a global discussion of mathematical instruction K-university levels.

Let us now examine the eight areas more closely.

(16) I. Mathematics (Itself)

It is easy to neglect mentioning mathematics itself. I was in the process of doing exactly that when Don Bushaw of Washington State University reminded me of it.

> First, I need to make the obvious remark that mathematics is one of the disciplines that have the advantage of being inherently international. There is no reason why mathematics should be culture-bound, and it is not. (Of course, there are pockets of folk mathematics and ethnomathematics all over the world, and some of these are extremely interesting and even illuminating. I am speaking of the "main stream.") I really wonder, however, how many people, having studied mathematics for years, really know this. Mathematics educators should probably lay more emphasis on this fact, thus providing their students with an illustration of the universal aspects of human thought and a paradigm for other possible styles of worldwide discourse that may now be obstructed by local or national barriers.

(17) Specific instances within mathematics are more easily seen. Numeration systems form an obvious one. For instance, the Cotocoli of northern Togo use multiples of 20 and 200 in designating numbers. From the identification of simple geometric forms to the classification of symmetry group patterns, art provides many examples. I exhibited a large collection of African artifacts at our county library during Afro-American Month this year. Featured were several objects from the Bushongo (Bakuba) who are particularly noted for their use of beautiful geometric patterns. Positive comments were made to the librarians by young and old alike. More indirectly, I have used examples from African languages when teaching elementary logic. My students initially have great difficulty with "or." I find it useful to give examples from Latin and French. Even more interesting to them is an example of a different kind of "or" from Otetela of central Zaire. "I am hungry 'kano' I am tired," means, "I am almost certainly hungry, but there is a slight chance I am tired." "Kano" is their only word for "or" and one can easily see the difficulties I had in teaching Batela students how to form the union of two sets!

II. History of Mathematics

(18) Most popular among my respondents was "history" as a vehicle for global awareness. In school mathematics, very little is done that was not developed outside our own country. Thus, any attempt to portray mathematics as a living, growing discipline must, at that level, necessarily refer to other cultures and countries.

Both NCTM Guidelines (5) and MAA Recommendations (6) call for an extensive understanding of the history of mathematics. In each and every of the four NCTM grade levels for teacher preparation -- early childhood and primary school

grades (let me call it G1 for Guidelines Level 1), upper elementary and middle school grades (G2), junior high grades (G3), and senior high grades (G4) -- there is an explicit statement that teachers should be able to "describe the historical and cultural significance of some of the mathematical principles usually taught in those grades."

(19) All of the first three levels call for this competency to be acquired within the courses listed for those levels. In Level G4, a separate course in the history of the mathematical sciences is listed. No specifics are given.

(20) By contrast, the MAA recommendations are very detailed in describing the contents of their recommended courses. Their levels of instruction will be denoted R1 for teachers of elementary school mathematics, R2 for elementary school mathematics specialists, coordinators of elementary school mathematics programs, and teachers of middle school and junior high school mathematics, and R3 for teachers of high school mathematics.

(21) For R1, four courses are recommended, the first two of which have as an objective, "Describe the historical and cultural development of some of the main mathematical concepts and principles usually taught in the elementary grades," while the third has, "Explain the role of geometry in our cultural heritage -- from both a scientific and an aesthetic viewpoint." For R2, the four R1 courses, an introduction to calculus, and four R3 courses (from a list of ten) are suggested. Five of those ten are labeled as being particularly appropriate, and "History of Mathematics" is one of them.

(22) Thirteen courses are listed for R3. Although lengthy, I now incorporate the five pages of objectives, topics, and references for the "history" course.

History of Mathematics

The history of mathematics is concerned with the origins, philosophy, and development of the mathematical sciences. The prerequisites are Calculus 1 and additional background acquired in a variety of other branches of mathematics.

Objectives

The objectives of the history of mathematics course are to provide teachers of mathematics with:

1. An understanding of mathematics both as a science and as an art (mathematics as a deductive science is emphasized in most mathematics courses; as an art, mathematics is a creative subject that includes the application of inductive insights and intellectual curiosity to the solution of problems and the formulation of theorems);
2. The ability to develop a broad concept of the mathematical sciences as approachable from several points of view including
   --problem solving, as a basis for the initial development of many concepts;
   --mathematics as a human endeavor, the role of individuals of both sexes with their insights and idiosyncrasies;
   --mathematics as a cultural heritage, the evolving role of mathematics in cultures throughout the world;
   --the impact of social, economic, and cultural forces on mathematical study and creativity;

--interrelations among the various branches of mathematics, especially their role in the solution of significant problems and in extending the horizons of mathematics; and

--the dynamic nature of mathematics, including the relatively recent development of probability and statistics and the increasing roles of calculators and computers;

3. Resources for developing the empirical and mathematical origins of each area of school mathematics including the notations, terminology, and major topics of algebra, geometry, trigonometry, calculus, number theory, probability, statistics, computer science and non-physical-science applications of mathematics. Such developments should be recognized as useful at all levels for organizing knowledge in historical perspective and appropriate in more detail for enrichment.

Topics

This history course should include, but not necessarily be structured according to, the chronological development of the mathematical sciences. The emphasis should be upon mathematical concepts and their interrelations. The suggested topics are a sample of the many available and worthwhile possibilities. The numbers of class hours are included to suggest approximate emphasis upon topics but are expected to vary according to the backgrounds and interests of the students.

. INFORMAL ORIGINS (4 hours)

Arithmetic and geometric concepts in early and primitive cultures, mathematical procedures based upon experiences and presented by examples,

early numeration systems, the nature and content of the Rhind Mathematical Papyrus and Babylonian cuneiform tablets, extensive uses of tables.

. THE EARLY DEVELOPMENT OF MATHEMATICS AS A SCIENCE (7 hours)

The work of the early Greek philosophers, the beginning of demonstrative geometry, the Pythagoreans, figurate numbers, commensurable and incommensurable magnitudes, Zeno's paradoxes, Eudoxus' method of exhaustion and theory of proportion, the three famous problems, geometric algebra, postulational thinking, Aristotle's laws and his concern for foundations, Euclid's Elements.

. MATHEMATICS IN THE GREEK CULTURE AFTER EUCLID (6 hours)

Continued efforts to understand the physical universe stimulate many aspects of the progress in mathematics. Note the continued development of geometry, properties of numbers, an early form of algebra, and the work of Archimedes, Eratosthenes, Apollonius, Hipparchus, Claudius Ptolemy, Heron, Diophantus, and Pappus.

. MATHEMATICS OUTSIDE EUROPE BEFORE 1600 (5 hours)

The mathematics developed in Chinese, Hindu, Arabic and other cultures. Suggested topics include rod numerals, the Chou-pei Suan Ching, the algebraic methods of Chu Shih-chieh, the Sulvasutras, the algebraic methods of Brahmagupta and Bhaskara, the algebra of al-Khowarizmi and Khayyam, and the Arabic contributions through the preservation, compilation, and extension of Greek and Hindu mathematics.

. MATHEMATICS IN EUROPE BEFORE 1600 (3 hours)

The introduction and transmission of Hindu-Arabic numerals, translations of Euclid's Elements and other manuscripts, development of

trigonometry, development of arithmetic and algebraic notations, solutions of cubic and quartic equations, slow acceptance of decimal fractions, and extensions of synthetic geometry.

. MATHEMATICS IN THE 17TH AND 18TH CENTURIES (7 hours)

Analysis emerges first as algebra and then as calculus. Algebra acquires a significant role. Geometry gives way to analysis as the leading area of activity. Suggested topics are logarithms, theory of equations, theory of numbers, probability, statistics, analytic and synthetic geometry, the boost from science, calculus, infinite series, differential equations, and widespread explorations some of which, such as operations with divergent series, involved highly theoretical extensions.

. MATHEMATICS IN THE 19TH AND 20TH CENTURIES (7 hours)

Non-Euclidean geometry sets the stage for abstract mathematical systems that do not necessarily describe the physical universe. Geometry and algebra evolve into abstract mathematical systems. Set theory and logical foundations are developed. Gradually there is an increased recognition of the importance of each area of mathematics as a useful approach to the study of the mathematical sciences. Statistical methods, the use of computers, and a continued emphasis upon applications are important aspects of the ongoing development of the mathematical sciences.

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The books on the history of mathematics by Boyer and Eves are widely used as texts. The NCTM Yearbook and the book by Kline do not have exercises. The NCTM Yearbook contains seven articles and 120 capsules on historical topics that are of special interest to teachers.

(23) As can be seen from this R3 "History" description, such a course would indeed promote "global awareness." In my opinion, it constitutes a "what should be" and "what is possible" facet of an international dimension to mathematics education.

(24) A great quantity of material seems to be available. From 1908-1965, for instance, the NCTM's Mathematics Teacher lists some 400 entries under "History of Mathematics." In the last decade, the pace seems to have slackened, but there are still several such articles and reviews in the Mathematics Teacher, as well as other journals. Required reading should be Jaffe's "Ordering the Universe: The Role of Mathematics" in the National Academy of Sciences/National Research Council's report on renewing U.S. Mathematics (7).

(25) Mention should be made of the International Study Group on the Relations Between History and Pedagogy of Mathematics (HPM), which is affiliated with ICMI. Their Americas Regional Affiliate has been meeting in conjunction with NCTM since 1982. They also publish a newsletter (8). A good contact person for further information about HPM is Bruce Meserve of the University of Vermont.

(26) In terms of "what is," I am not so optimistic. In several universities, no history of mathematics course exists. In those that do, a course in the mathematics department with several math majors might tend to emphasize more recent developments; i.e., those of least interest to the future teacher. I do know, however, that some mathematics educators offer a history course within the college of education, sometimes with a very clever title. Again, an external force demanding an "international dimension" across the board in teacher education would be extremely helpful.

(27) I believe an appropriate course in the history of mathematics for prospective teachers is the best bet for promoting global awareness among the preservice group.

III. Comparative Studies

(28) For lack of a better term, let me use this to refer to certain aspects related to knowledge of mathematics education in other countries. "What is" appears to be highly non-uniform and depends on the size and prestige of the university and on the backgrounds of individual mathematics educators. The following fall into the "should be/possible" categories:

(29) A. Journals from other countries. Because language is a problem, I would suggest Mathematics Teaching (Great Britain) which is similar to NCTM's Mathematics Teacher. Educational Studies in Mathematics and perhaps L'Enseigement Mathematique are other possibilities. On a recent trip to the library I saw interesting articles from these respective journals on problems about math syllabi in England (9), geometry in English secondary schools (10), and a 1966 article by Piaget (11). Whenever students are assigned articles from our domestic journals, perhaps related articles from foreign journals could also be used. Math Teaching is not an expensive journal.

(30) B. ICMI Materials. This is similar to "A" and would include the quadrennial "Proceedings" and abstracts of the poster sessions and short communications. ICMI also publishes a bulletin. For example, recent issues have contained articles on "language and mathematics." (From the UNESCO Nairobi Conference through the most recent of ICMEs, much has been written about it.) Other materials are available from groups who were a part of ICME. For instance, Don Dessart of the University of Tennessee wrote that the "Using Research Group" has produced a series of about 25 international papers that will soon be published by the Wisconsin Center for Education Research.

(31) C. Second Internatinal Mathematics Study Group. In conjunction with the ERIC Center at Ohio State, they have produced international reviews of issues in mathematics education in four areas: calculators, minimal competencies, applications and women and mathematics. A good contact is Ken Travers of

the University of Illinois. Hopefully, the Second International Study of Mathematics will receive more attention by undergraduates and teachers than the first one did.

(32) D. Mathematics curricula elsewhere. One could study either a specific country or a whole group of countries. One could study claims about the quality of foreign education.

(33) E. Foreign visitors, students and faculty.. Their use could range from describing school mathematics (algorithms, how they were taught, etc.) by students and faculty from other disciplines based on their own school experiences to highly technical lectures by visiting mathematics educators.

(34) F. American math educators travel abroad. Attendance and participation in at least one ICME could be expected, or at least encouraged, for faculty. Unfortunately, as Chairman of the joint NCTM-MAA NSF Travel Awards Committee for ICME-5, I saw indications that several universities were very reluctant to be of financial assistance to their faculty. Likewise, faculty members could be encouraged to participate in study tours to other countries. In the past decade, groups of American math educators have traveled to China, Cuba and the Soviet Union.

(35) G. Our professional publications. The Mathematics Teacher lists some 60 articles under "Mathematics in Other Countries" for the period 1908-1965. There have been some articles in recent years, although not as many as some of us would like. Other examples abound. The NCTM's Journal for Research in Mathematics Education had a report last year of the International Study Group on Minimal Competencies in Mathematics. The MAA journals occasionally have such articles. A book was published by MAA about a recent study tour of China.

IV. Word Problems and Applications

(36) To me an obvious place to promote global awareness in our schools is through the use of word problems. On a trivial level, a problem such as comparing the direct distance Los Angeles - New York versus LA - Chicago - NY can be changed to New York - Dakar versus via Paris. Yet, as Dubisch (letter attached) correctly points out, children are taught to ignore the extraneous details (in this case, city names). Thus, the impact might not be as great as I had hoped. Some other math educators are not overly impressed with this idea either.

(37) On a deeper level, it seems that meaningful problems based on real situations might convey a feeling for life in other countries. For instance, the incredible hardships of students attending a boarding school in the remote interior of the Congo when I taught there in the mid-sixties might give our students a better perspective of the value of an education.

(38) Frank Swetz, in the March 1985 Math Teacher (12), writes:

> . Every day of the year 40 000 children throughout the world die from malnutrition and infection.
>
> . Women and girls compose half the world's population, work two-thirds of the work hours, receive a tenth of the world's income, and own a hundredth of the world's property.
>
> The impact of these messages is far more effective than that of merely noting that "hunger is a problem" and "women are discriminated against." The numerical facts, the magnitudes, help convey the degree of the injustice in question. Furthermore, the exploration of numerical relationships among peoples and countries can be used as a channel of human understanding, one that transcends cultural and national boundaries, and as a clarifier of the common problems that face all men and women.
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> . . .
>
> The collection and use of the data in making cross-cultural comparisons should not result in a "them and us" mentality but rather should focus on "we." Situational discussions must emphasize the mutual responsibility of nations and peoples in enduring and solving the universal problems that confront us. The use of global perspectives in mathematics teaching, while assisting in the development of cognitive skills, also supplies a basis for affective growth. By using mathematics to explore issues of fundamental importance to humans, it is hoped that students will develop attitudes that transcend parochial biases and reflect their understanding of the interdependence of institutions, nations, and peoples. Thus, mathematics and its teaching are truly vehicles for global awareness.

(39) V. Metrification. This area needs no explanation.

(40) VI. Mathematically-based games. Of interest to all students, but perhaps particularly appropriate for children in the lower grades and their teachers are certain games from around the world. Variations of the African game of "Mancala" come to mind (See Zaslavsky (13)). A good contact is Barnabas Hughes, O.F.M., of California State University in Northridge. He developed such a course eight years ago and teaches it annually for one unit in the mathematics department.

(41) VII. Mathematical contests. Of interest to math educators, school students and their teachers should be problems from various math contests around the world. Working such problems is not only good mathematics, but can also lead to a discussion of the curriculum and an appreciation of the standards of excellence in other countries. The World Federation of National Mathematics Competitions has just published its first newsletter (14). A good contact is Walter Mientka of the Unviersity of Nebraska. He is one of the Federation's Vice Presidents.

(42) VIII. Computers and technology? In this rapidly expanding area it would clearly be advantageous for math educators and school teachers to be aware of what is being done in Europe and Japan. ICMI's seminar on "The Impact of Computers and Informatics on Mathematics and its Teaching," which will be held this spring in Strasbourg, comes to mind.

(43) The eight areas above contain some of the ideas advanced by individual colleagues and me. I have tried to mainly stick to the "what is possible" as opposed to "what should be." For an example of the latter, see the course suggestion of Henry Pollak of Bell Communications Research in the attached letter. If one is to proceed further it will be necessary to identify a corps of knowledgeable persons who can quickly point interested parties in promising directions. I have already mentioned Bushaw, Dessart, Hughes, Kilpatrick, Meserve, Mientka, Pollak and Travers. For their

specific interest in mathematics education in other countries, let me emphasize these persons: Claudia Zaslavsky (45 Fairview Avenue, New York, NY 10040), Roy Dubisch University of Washington), and Frank Swetz (Penn State University - Middletown). John Dossey of Illinois State University and Don Hight of Pittsburg (KS) State University have both been involved with the guidelines and recommendations for teacher training programs. World class math educators whose professional endeavors have international connections include Shirley Hill (University of Missouri - Kansas City) and Jeremy Kilpatrick (University of Georgia). As an excellent classroom teacher, I think of 1984 Presidential Award winner LeRoy Dalton (Wauwatosa West HS, Wauwatosa, Wi). Close to ICMI are A. G. Howson (University of Southampton, Centre for Mathematics Education, Southampton, S09 5NH, United Kingdom), B. F. Nebres (Ateneo de Manila University, P. O. Box 154, Manila, Republic of the Philippines), and Ubiratan D'Ambrosio (Coordenador Geral dos Institutes, Universidade Estadual de Campinas, CP 1170, 13100 Campinas - SP - Brazil).

(44) Where do we go from here? If an international dimension to mathematics education is perceived by the membership to be needed, I am certain our professional societies would respond. An NCTM or joint NCTM/MAA Committee could be appointed in which case external funding would be greatly appreciated. A careful search could be conducted both here and abroad. Perhaps MAA or NCTM could publish a book of readings on international math education with articles from our journals. An NCTM yearbook or a book in the MAA Notes Series could be devoted to a collection of papers on international math education by writers from around the world. We could encourage more papers at our annual meetings and those of the affiliates. Receptions for foreign visitors could occur at annual meetings. We could strengthen ties with NCTM's corresponding societies and extend the list. We could establish closer ties with the Peace Corps, USIA and AID. We could publish 80-90 word capsule reviews of selected articles from foreign journals. NCTM did so in the past using 30 journals from 24 countries in 17 languages. I am certain that this list of ideas would be only a starting point for the committee suggested above.

(45) In closing, let me share an excerpt from the letter of a friend. Still smarting from his latest encounter with a reviewer for one of our professional journals, he wrote, "Well, good luck, and let me know if you have any success. Perhaps I will live long enough to participate in some of the changes in math ed along the lines you indicate!" I am optimistic. The ICMEs are functioning better than ever, several international interest groups are increasing the ties of individuals across international boundaries, greater numbers of foreign visitors are arriving (e.g., the impressive list attending the March University of Chicago School Mathematics Project Conference), and the American government is involved in mathematics education projects in the Third World (e.g., the U.S. Information Agency's Teacher-Text-Technology Initiative involving five African countries).

(46) Vox clamantis in deserto? No!!!

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2. Hill, Donald. "Peace Corps Training of Mathematics/Science Teachers," Conference Board of the Mathematical Sciences Newsletter, Vol. 12, No. 4, October-November, 1977, pp. 53-55.

3. Hill, Donald. "Mathematics Legislation in Florida (USA)," International Commission on Mathematical Instruction Bulletin, No. 17, December, 1984, pp. 28-29.

4. Page, Warren, ed. American Perspectives on the Fifth International Congress on Mathematical Education, Washington, D.C., Mathematical Association of America, 1985, p. v.

5. Guidelines for the Preparation of Teachers of Mathematics, Reston, VA, National Council of Teachers of Mathematics, 1981.

6. Recommendations on the Mathematical Preparation of Teachers, Washington, D.C., Mathematical Association of America, 1983.

7. Jaffe, Arthur. "Ordering the Universe: The Role of Mathematics," in Renewing U.S. Mathematics, Washington, D.C., National Academy Press, 1984, pp. 117-162.

8. International Study Group on the Relations Between History and the Pedagogy of Mathematics Newsletter, No. 7, October, 1984, Editor Charles V. Jones, Ball State University, Muncie, IN, 47306.

9. Knowles, Frank. "Core Syllabuses and A-Level Mathematics," Mathematics Teaching, No. 90, March, 1980, pp. 40-43.

10. Lang, Bryan and Peter Ruane. "Geometry in English Secondary Schools," Educational Studies in Mathematics, Vol. 12, 1981, pp. 123-132.

11. Piaget, Jean. "L'initiation aux mathematiques, les mathematiques modernes et la psychologie de l'enfant," L'Enseignement Mathematiques, II Series, Vol. 12, Oct.-Dec., 1966, pp. 289-292.

12. Swetz, Frank. "Mathematics: A Vehicle for Better Global Understanding," Mathematics Teacher, Vol. 78, No. 3, March, 1985, pp. 207-215.

13. Zaslavsky, Claudia. Africa Counts: Number and Pattern in African Culture, Boston, Prindle, Weber, and Schmidt, 1973, pp. 116-136.

14. World Federation of National Mathematics Competitions Newsletter, No. 1, December, 1984, Editor Warren Atkins, C/-Canberra C.A.E., P.O. Box 1, Belconnen, A.C.T. 2616, Australia.

Dear Don:

In your recent letter, you identified several obvious ways in which the teacher education curriculum in mathematics might do more to make undergraduates more aware of other nations and cultures. I think opportunities to explore the school mathematics curriculum in other countries provide the best avenue to that awareness, although I recognize that such opportunities are limited in most undergraduate programs. May I suggest the book Curriculum Development in Mathematics, by Geoffrey Howson, Christine Keitel, and me, published by Cambridge University Press, as a source of information on curriculum development work in several countries. Some of this material might be included in the teacher education curriculum. One of the IDM publications, edited by Hans Steiner and entitled Comparative Studies of Mathematics Curricula, is also a good source.

Undergraduates in teacher education can get a better sense of how people in other cultures do mathematics, as well as gaining a better understanding of the nature of algorithms, by learning the algorithms taught in other countries for operations such as subtraction and division. Foreign students or faculty in the university might be invited into methods courses to demonstrate the algorithms they were taught. I think it would be especially good for elementary preservice teachers to see how an abacus is used by an expert and how various finger patterns are used in oriential countries to learn number combinations.

Here at Georgia, we invite foreign visitors to our department to give colloquia on mathematics education in their countries. We encourage our undergraduates to join the student organization (the Mathematics Education Student Association, an NCTM affiliate), which sponsors the colloquia, but the undergraduates' schedules do not always permit them to attend. We could probably schedule classes better so that more of them could attend. The students do appear to be interested in how mathematics instruction is organized and conducted in other countries, what the role and status of teachers are, etc.

I don't see international settings for word problems as offering much, but I agree that the history of mathematics is an obvious avenue for the awareness you seek. Our history of mathematics course is popular with undergraduates, and it does, I think, make them more aware of the contributions to mathematics of people from other cultures.

Finally, my I draw your attention to the special issue of JRME in March 1984 on minorities and mathematics, and in particular, the articles by Bradley, Cheek, Tsang, Valverde, and Cuevas. Although there is little attention in these articles to minority groups outside the United States, there are some useful ideas about discussions of minority cultures within the United States--groups that should not be overlooked as guidelines are set up for enhancing international awareness.

I hope these thoughts are helpful to you. Good luck in your work. See you in San Antonio, if not before.

Cordially,

Jeremy

Jeremy Kilpatrick

(2/19/85)

Dear Don,

You have set yourself a most difficult task in "internationalizing" teacher training in mathematics and I'm afraid that I can't be of much assistance to you. But here are my thoughts:
(1) History of math. is, as you say, an obvious place to internationalize. However, lacking in most histories is any account of early developments in, for example, Africa. (See remarks in Zalevsky's book on this.) Futhermore, prospective elementary teachers are very unlikely to take such a course and it is not always even offered in colleges and universities.
(2) I would agree that someplace in the curriculum (both for prospective elementary and prospective secondary teachers) there should be some discussion of math. curricula in other countries (including some discussion of over-all curricula, grade organization, standards, monies available, etc.). However, I feel that discussing just one country in detail would not be the best approach. Difficult as the task might be, I think that there should be some comparisons, grouping together countries with a similar curriculum and educational philosophy and resources even tho vast over-simplifications would need to be made in such an approach (e.g., Western Europe, Soviet bloc, African, S. American). Attention should also be made to different methods of preparing teachers elsewhere -- teacher training colleges, Dip.Ed. etc.
(3) I can't really see any way of internationalizing word problems at the level of mathemtics involved in teacher education except, possibly, in courses for prospective elementary school teachers. Even here (and the following remarks would also apply to the pre-college curriculum) I doubt whether textbook authors and publishers could be persuaded to include such problems. Furthermore, I believe that the effect on students encountering problems containing "foreign" data would be very minimal. This is because, very properly, we have taught students to ignore non-significant data in a problem. Thus, for example, a problem which asks a student to compute the per capita expenditure for education in Ghana, given the budget and school population figures, is likely to be done and the answer recorded without the significance of the low figure obtained being realized. (This is not to argue against the inclusion of such problems. And certainly an internationally minded instructor can use such problems as the basis for a discussion of third world educational financing difficulties.)
(4) Obviously, no real progress is possible without the cooperation of the teacher training institution instructors. Hopefully, the report you are contributing to will assist in gaining this cooperation.
(5) Having commented on your suggestions I should certainly add some of my own! Alas, even after spending considerable time meditating on the problem, I am unable to come up with additional ideas. The real problem lies deep within our culture. Except when it is a matter of floods, famines, war, etc. or when an event has a *direct* influence on the U.S.A., few people pay attention to news about other countries. (In fact, it is difficult for the average person to find sources of international news beyond the categories I have mentioned!) Schools at all levels can help to change such attitudes but it is a most difficult task: we can only try our best.

...

Roy Dubisch

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Dr. Don Hill
3017 Godfrey Place
Tallahassee, Florida 32308

Dear Don,

This is in response to your letter on the international dimensions of undergraduate teacher education. You have named a number of important areas which should be covered; let me enlarge the problem a bit, philosophize further, and then make some recommendations.

I should first like to call attention to Ubi D'Ambrosio's plenary session paper in Adelaide. He spoke of the difference between ethno-mathematics and learned mathematics. What he meant by ethno-mathematics really became clear through the examples in the latter part of his talk: In the weaving, the canoes, the huts, the pictures of the particular Indian culture, there is a lot of mathematics, and thus a lot of common mathematical culture is shared among the tribe. It is not the mathematics learned in school, whose usefulness is often far from apparent and which is typically not connected to the life and society of the individual. It would be very important for prospective teachers to be aware of this kind of problem. Get a copy of Ubi's paper, ask him for related material, and work that sequence through.

This brings me to the larger topic of mathematics and language. There was a conference on the subject in Nairobi probably over 10 years ago. It was excellent, Unesco published a volume of the papers which Ed Jacobsen should be able to get for you, and that is very important material. The subject was a major topic at the Berkeley Congress; see Chapter 16 of the Proceedings for the breadth of the ideas involved.

I think another main topic is that of the curriculum and its dependence on time and location. There is no such thing as the right mathematics curriculum independently of which country - or part of the country - you are in. It must fit with the society from which the children come. A great mistake in the 1960's worldwide was to try to export curricular reform materials as such from the USA. Exporting the spirit and zeal was fine, but the examples and the applications and the particular emphases have to be local. You know this better than I, by far; you have lived that kind of a problem; but it should be part of what you are planning. I'll never forget the consternation in the girls' secondary school in Morogoro when the problem on heading in the Cambridge-Oxford exam - the girls had drilled on this, and everyone of them could have done the problem - was tried by NO student. Why not? Because it was written in terms of a roundabout, and nobody knew what that meant!

What it all adds up to, I guess, is a course on the relationship of mathematics to the society in which you live - to its language, its problems, its culture. ...

H. O. Pollak
(2/21/85)

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